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Kinematic Analysis of a Robotic Manipulator

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Abstract— The kinematics of a robotic manipulator is discussed in this paper. The joint angles required to obtain the particular position and orientation of the end effector are determined using Inverse Kinematics. The paper analyses this method. The transformation between the base frame and the end effector frame is found out to solve the inverse kinematics. Further Jacobian is used to find the relationship between the joint velocities and the end effector velocities. Also the relation between the joint torques and the end effector forces can be found from the Jacobian. The trajectory generation, different types of curves of the trajectory and the trajectory planning with obstacles are also discussed.

Index Terms—End Effector, Euler Angle, Humanoid Robot, Inverse Kinematics, Jacobian, Manipulator, Trajectory Generation.

1 INTRODUCTION

Determining the position of the robot and its different bodies is an important task while designing a humanoid robot. For this purpose GPS, encoders, vision system etc. can be used. Once the robot is located it is critical to find where the hand is positioned, where the leg is positioned and so on. Finding the position and orientation of the end effector with respect to base by using the known joint angles is forward kinematics. It is a mapping from the joint space of the links to the Cartesian space of the end effector. Finding the joint angles for the desired position and orientation of the end effector with respect to base is inverse kinematics. It is going from Cartesian space of the end effector to the joint space of the links. Solving a forward kinematics problem is domain independent, while an inverse kinematics problem is domain dependent.

Two conditions for obtaining a closed form joint solution of a robot arm in which either three adjacent joint axes are parallel to one another or they intersect at a single point are mentioned in [1]. Although a robot arm may satisfy one of these two conditions for finding the closed form solution, it is difficult to build a consistent procedure for it [2]. A closed form joint space solution is obtained for a 6-DOF humanoid robot arm in [3]. The kinematics of a 7-DOF redundant arm of the humanoid robot ARMAR is described in [4]. To obtain a closed for solution they used the constraint on the elbow position and divided the problem into smaller sub-problems. The geometric perception is used to solve the system of equation using geometric method [5], [6]. But it may become hard to obtain the solution when more than four joints are involved. In this paper a simple and efficient technique to develop the kinematic model of a robot manipulator is discussed.

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2 TRANSFORMATION

The basic manipulator is shown in Fig.1. There are different links between the base and the end effector which will carry the end effector to move to different locations.

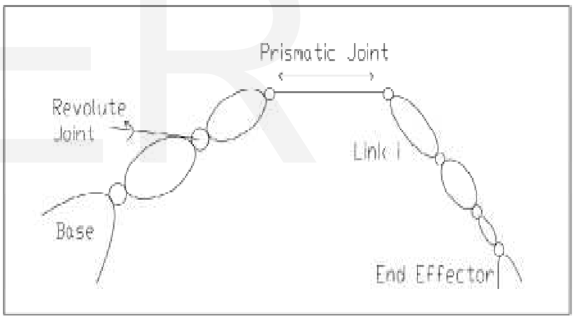

Fig. 1: Manipulator

Two links are connected by a joint. Any set of joints can be reduced to revolute joint and prismatic joint. Each of them has one DOF. Revolute joint allows the rotation about a fixed axis and prismatic joint allows the translation about a fixed axis. In a manipulator having n moving links and one fixed link, there are n joints and hence n degrees of freedom. Let m denote the number of configuration parameters of an end effector. If n>m then the robot is redundant.

Degree of redundancies= n-m.

In this case if a robot arm is to be moved to a particular position then there could be many configurations to reach that position. The existence of multiple solutions makes the problem of inverse kinematics very difficult to solve. But redundancies are useful to avoid the obstacles.

To solve the problem of the inverse kinematics it is essential to obtain the position and the orientation of the robot arm itself in the space. The position and orientation of the base are known. Hence, the relationship between the base frame and the end effector frame is determined. To determine this

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relationship the transformation between different joints and finally the total transformation matrix is to be found.
As shown in Fig. 2 $^{(i-1)}T_i$ is the homogeneous transformation matrix between frame $R_{(i-1)}$ and R_i which takes into account the rotation of the frame R_i with respect to frame $R_{(i-1)}$ and the translation of the origin of frame R_i with respect to origin of frame $R_{(i-1)}$.

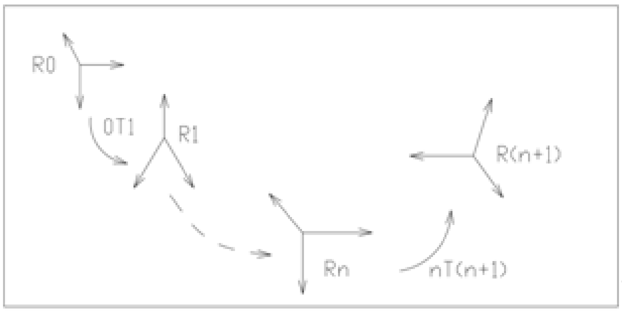

Fig. 2: Transformation among frames

The final transformation matrix from the base to the end effector is derived as

$$T= {}^{0}T_1 {}^{1}T_2 {}^{2}T_3 ,\ldots\ldots\ldots {}^{n}T_{(n+1)} \quad (1)$$

This 4×4 transformation matrix (T) is an ortho-normal matrix and thus, $T^T=T^{-1}$

3 End Effector Configuration Parameters

The end effector configuration parameters are given as:

$$\begin{bmatrix} X_P \\ X_R \end{bmatrix}$$

Where X_P is the position representation and X_R is the orientation representation

3.1 Position Representation

X_P can be found out by the first 3 elements of the last column of the matrix (T) and are called the translational elements.

3.2 Rotation representation

Rotation representation X_R can be found out by the first 3 rows and 3 columns of the matrix (T) and are called the rotational elements. There are several ways to find this representation. In Euler angle representation X_R is given by successive rotation about x, y and z axes by θx, θy, θz respectively.

$$R=RzRyRx \quad (2)$$

This Euler angle representation is minimal. But this method faces a problem of singularity for cos θy =0 or θy =90. This shows that for some mathematical representation the configuration may fail and the arm gets locked. In the region around the singularity a larger force is needed to be applied to reach a particular point. This problem can be eliminated by using four Euler parameters in the 4D space.

4 Link Description

Each link has to be given a frame to find the relationship between the end effector frame and base frame. The most obvious way to assign a frame is to put it at the centre of mass of each link. Then the transformation matrix can be found. But each link joint has only 1 DOF. So to take the advantage of the constraints and the DOF, the frames are not randomly put at the centre of mass. Instead, Denavit–Hartenberg parameters are used.

4.1 Denavit-Hatenberg Notation (D-H parameters)

This is a minimal set of parameters to represent the relationship between 2 successive links on a chain. This notation and parameters are used to describe the forward kinematics.

As shown in the Fig. 3, the normal distance between 2 link axes is a_{i-1}. It is the link length. The orientation between those links is α_{i-1}. These 'a' and 'α' are constants. This is the link description. The angle between two normals between successive links is θ, whereas offset between these two normals is d.

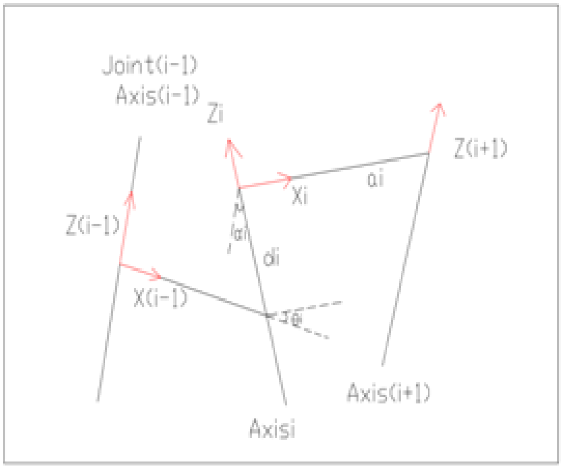

Fig. 3: Frame Attachment

If the joint is revolute, then d is constant and θ is variable. If the joint is prismatic, d is variable and θ is constant. The propagation between the frames is done using the four D-H parameters a,α,d and θ. Three of them are constant and one is variable; either d or θ, which is obtained from encoder.

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4.2 Frame attachment

The frame attachment is done as follows:

1. Z axis: along the joint axis
2. X axis: along common normal
3. Y axis: according to the RH frame
4. Origin: at the intersection of the axis and the common normal

To reduce the complexity while attaching the frames to the first and the last links, it tried to make $a_0=a_n=\alpha_0=\alpha_n=0$ and to keep the constant parameters among θ_1, θ_n, d_1, d_n zero.
The transformation matrix between $z_{(i-1)}$ and z_i is obtained by the transformation matrix ${}^{(i-1)}T_i$ which is obtained by 4 D-H parameters $a_{(i-1)}$, $\alpha_{(i-1)}$, d_i and θ_i. Finally the total transformation matrix from the base to the end effector is determined by (1). This transformation matrix is used to find the end effector configuration parameters.

5 Jacobian

Once the geometry of a manipulator is known then the linear, angular velocities of the end effector, the torques applied at the joints and the forces and the moments resulting at the end effector can be found. Jacobian is used for that purpose. It gives 2 things:

1. Relation between velocities of the joint and the end effector velocities.
2. Relation between torques required at the joint motors and the forces obtained at the end effector.

If x represents the position and orientation of the end effector then

$$x = f(q) \tag{3}$$

where $q = \bar{\varepsilon}\times\theta+\varepsilon\times d$

$\varepsilon=0$ for revolute joint
$\varepsilon=1$ for prismatic joint

$$\begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} f(q)_1 \\ \vdots \\ f(q)_m \end{bmatrix} \tag{4}$$

$$\delta x_1 = (\partial f_1/\partial q_1)\,\delta q_1 + \dots\dots\dots(\partial f_1/\partial q_m)\,\delta q_m$$
$$\vdots$$
$$\delta x_m = (\partial f_m/\partial q_1)\,\delta q_1 + \dots\dots(\partial f_m/\partial q_m)\,\delta q_m \tag{5}$$

$$\delta x = \begin{bmatrix} (\partial f1/\partial q1) & \cdots & (\partial f1/\partial qm) \\ \vdots & \ddots & \vdots \\ (\partial fm/\partial q1) & \cdots & (\partial fm/\partial qm) \end{bmatrix} \delta q \tag{6}$$

$$\delta x_{(m*1)} = J_{(m*n)}(q)\ \delta q_{(n*1)} \tag{7}$$

$$\dot{x}_{(m*1)} = J_{(m*n)}(q)\,\dot{q}_{(n*1)} \tag{8}$$

Equation 8 gives the relationship between the velocities at the joint space and the velocities at the end effector.
The required $\delta\theta$ can be found out for the particular δx by the inverse of Jacobian. Differentiation of x with respect to q is carried out to get the Jacobian matrix. Similarly, the torque required at the joints to get the particular force at the end effector can be obtained using the Jacobian matrix.

6 Trajectory Generation

The trajectory generation by a robotic manipulator to go to the desired point from the initial point is shown in Fig. 4.
The basic problem of the trajectory generation is to move the manipulator arm from some initial position A to the final position C. The intermediate position B may be required to accomplish some tasks.

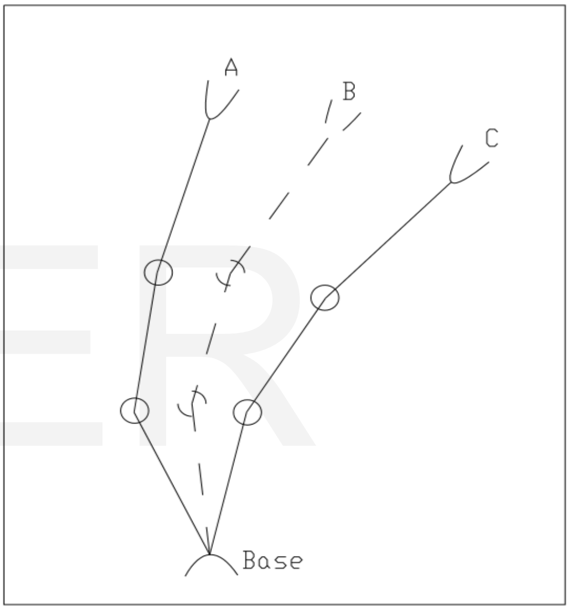

Fig. 4: Trajectory Generation

6.1 Solution spaces

1. Joint Space: In joint space the target co-ordinates are known. The inverse kinematics is solved to find the co-ordinate for each joint. Then by joining this target point to the initial point the trajectory is created. This does not face the problem of singularity. Calculations are less. But the straight line path can't be assured.

2. Cartesian space: In Cartesian space the co-ordinates in Cartesian space are known and a required shape can be tracked. One needs to take sample points and check if the robot is following the desired path. Hence, it is more expensive at run time.

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6.2 Candidate curves

1. Straight line

Fig. 5 shows a straight line curve. While going from A to D via B and C one can't guarantee that the velocity at B for path B to C is the same for path A to B.

Hence, there is a jerk at B. If one follows the path A to B, stops at B and then continues for B to C then there is no such problem. But stopping at intermediate points will waste the energy.

2. Straight lines with blends

Fig. 6 shows a straight line curve with blends. The curves are created around the intermediate points to avoid these jerks.

3. Cubic polynomials (spines)

Fig. 7 shows a cubic polynomial curve. It consists of 4 coefficients. Hence, maximum 4 constraints can be satisfied; for example initial point, initial velocity, final point, final velocity. Higher order curves are required to satisfy more constraints like acceleration. More calculations are required for this type of curve. Equations are found for each curve using the given constraints.

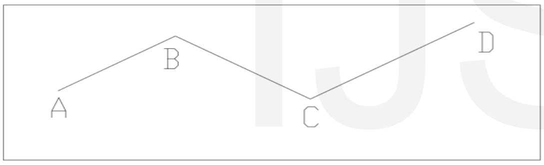

Fig. 5: Straight line curve

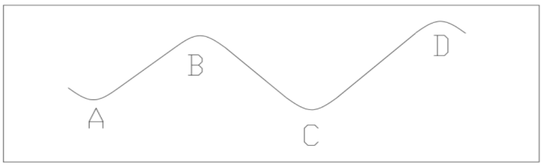

Fig. 6: Straight lines with blends

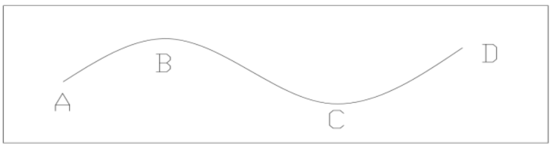

Fig. 7: Cubic polynomials (spines)

7 Trajectory Planning With Obstacle

7.1 Local versus global motion planning

Local planning is just the planning with the end effector. Global planning is the planning with the whole manipulator. Generally the combination of these two planning is taken. Global planning is used when there is relatively empty space and then one can switch to local planning when the space is full with obstacles.

7.2 Artificial potential field method

In Artificial potential field method the robot is considered to be a pole and the obstacles are considered to be of the same pole. Hence, there is repulsion between the robot and the obstacles and the robot follows the path avoiding the obstacles.

8 Conclusions

The kinematics of a robotic manipulator is discussed in this paper. To get the particular position and orientation of an end effector the joint angles are determined using inverse kinematics. This method is discussed in this paper. The transformation from the base of the manipulator to end effector is determined to solve the inverse kinematics. Jacobian is used to find the relationship between the joint velocities and the end effector velocities. It is also used to find the relation between the joint torques and the end effector forces. Finally, the trajectory generation, different types of curves of the trajectory and the trajectory planning with obstacles are discussed.

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